Solutions to

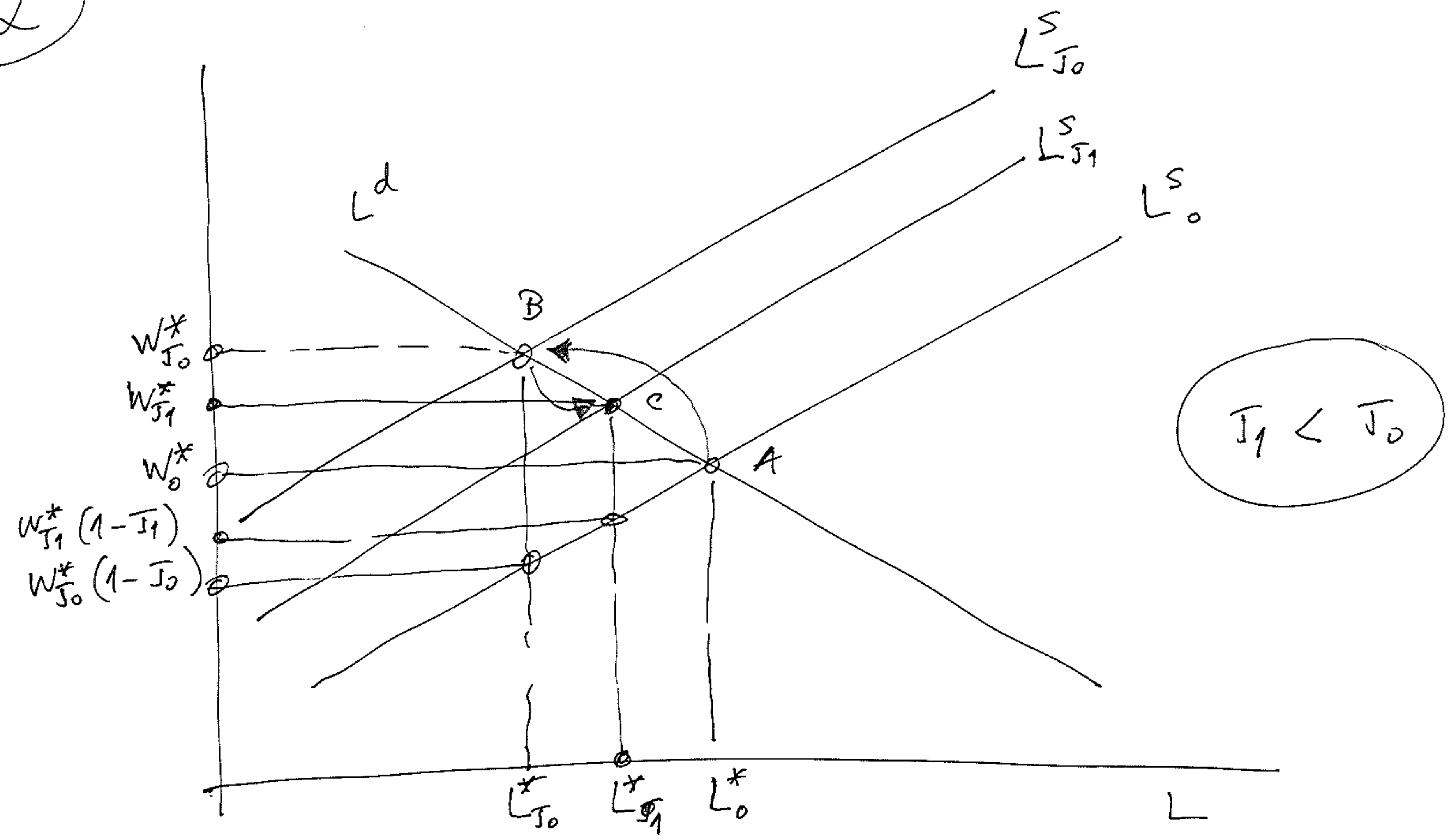
chapter 7 exercises

(2, 3, 4, 5, 6 and 8)

Please notice that these solutions may have one typo or another. If you spot one, please give feedback.

Vivaldo Mendes

2

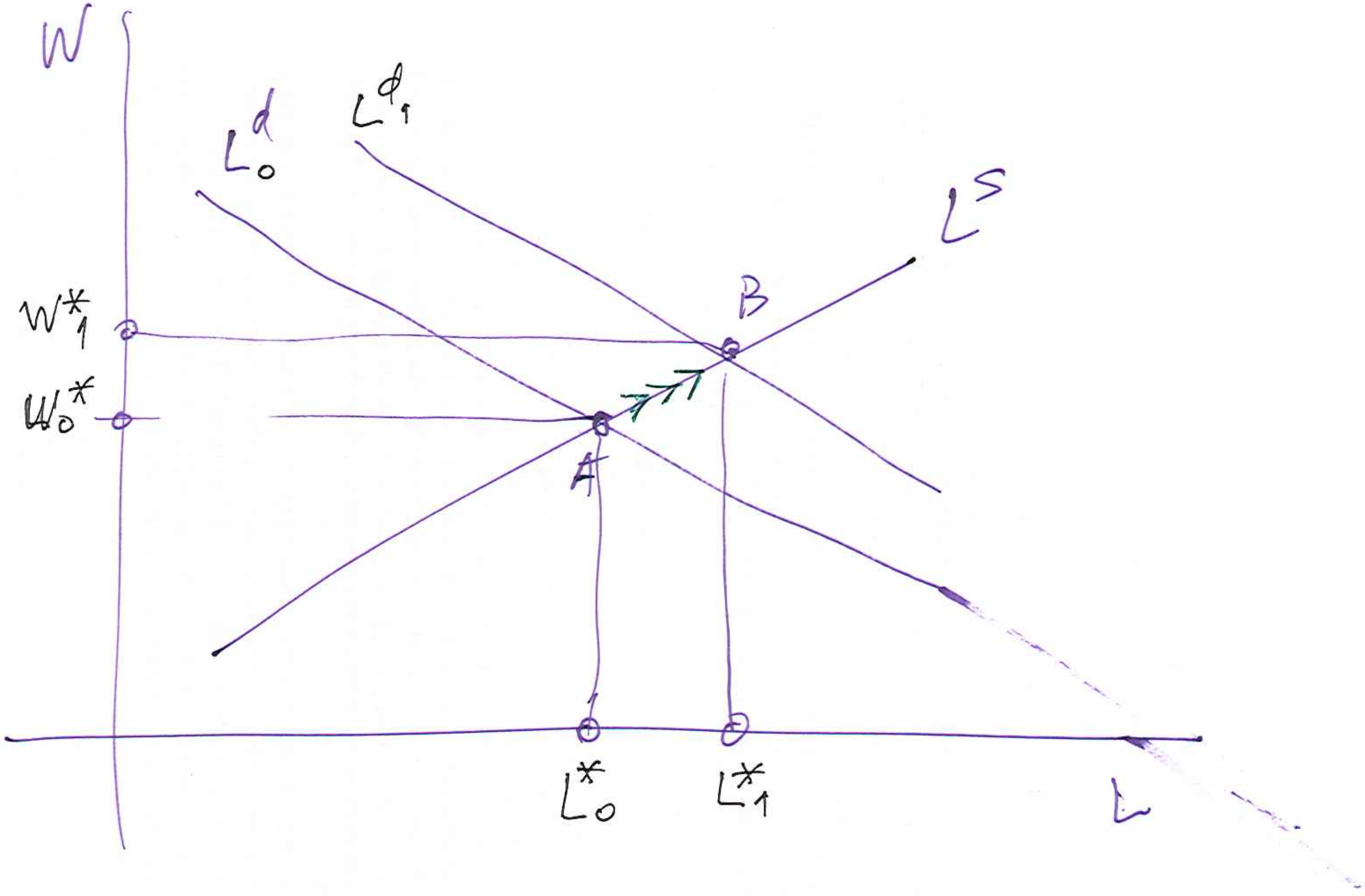


A : No taxes

B : Taxes at the level of J_0

C : Taxes at the level of J_1

3



The reduction in the price of oil leads to a reduction in production costs, which leads in turn to an increase in the demand for labor services.

4

In order to solve all questions in exercise 4, you need to recall some basic properties from geometric series. Let's recall them here:

(i) If you have a series with a common ratio "a" and allow time to go to ∞, then its solution is given by

$$s = x.a^0 + x.a^1 + x.a^2 + \dots + xa^n \qquad [1]$$

$$s = \frac{\text{1st term}}{1 - \text{common ratio}} = \frac{x}{1-a} \qquad [2]$$

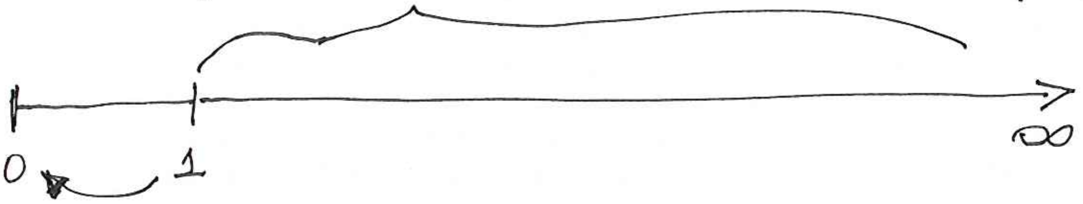
which is valid if and only if $|a| < 1$, $n \to \infty$.

(ii). Now, imagine that n is rather small, let's say 40 or 50, or 100. Then, the solution is given as you find it in the textbook

$$s = x\left[\frac{1 - a^{n+1}}{1-a}\right]. \qquad [3]$$

(Biii) You may have a different situation. The geometric sum starts only <u>next year</u>, and goes to infinity, but you want to know its Present Discounted Value <u>today</u>.

$$S = x.a^0 + x.a^1 + \dots + x a^n = \frac{x}{1-a}$$



The solution is simple: discount the sum <u>S</u> one further period backwards by doing:

$$S \times a = \frac{x}{1-a} \times a \qquad [4]$$

Now it is easy to confirm the solutions provided for exercise 4 at the end of chapter 7.

<u>4(a) and 4(b)</u> are easy to understand the solutions given in the textbook.

4(c) You have just to apply equation [2] to obtain the solution to the Present Discounted value

$$PDV = \frac{100}{1 - \underbrace{\left(\frac{1}{1+0.03}\right)}_{a}} = 3433.$$

4(d) You have just to apply equation [4] to obtain the solution to the PDV

$$PDV = \frac{100}{1 - \underbrace{\left(\frac{1}{1+0.03}\right)}_{a}} \times \underbrace{\left(\frac{1}{1+0.03}\right)}_{a} = 3333.$$

4(e) You have just to apply equation [3] to get the solution to this question

$$PDV = 100\left[\frac{1 - \overbrace{\left(\frac{1}{1+0.03}\right)}^{a}{}^{50}}{1 - \left(\frac{1}{1+0.03}\right)}\right]$$

$$= 2650.$$

Exercise 5

You are just asked to redo exercise 4, but now with two different discount rates:

1% and 5%

The answers are as follows (we insert the values for 3% as well:

	1%	3%	5%
a	49405	48544	47619
b	45264	37205	30696
c	10100	3433	2100
d	3958	3333	1917
e	3950	2650	1917

As a general comment, notice that if the discount rate increases, the PDV goes down.

Exercise 6

There is nothing new in this exercise. It asks you to redo the calculation of human capital as in section 7.5, but now with some different discount rates:

1%, 2%, 4% and 5%.

You will remember that with 3% we obtained the value of human capital equal to 1.41 millions. See the table for answers to 6(a):

1%	2.296 millions
2%	1.895 "
3%	1.41 "
4%	1.357 "
5%	1.175 "

6.(b) The economic intuition is simple: whenever you increase the discount rate, you are just saying that the present is more important (valuable) to you than the future, and so the same stream of income over the future will have a lower ~~value~~ value for you in the present.

Note: can you imagine what this kind of attitude towards the future will lead to as far as natural resources are concerned? Or what kind of impact will this ~~kind~~ type of behavior have upon the wellfare of future generations?

Exercise 8

This is an excellent exercise because it puts everything we have been dealing with together. And you have to be clever to come up with an answer.

(a) We will assume that :

- a college educated worker will work for 45 years
- therefore a non-college education will mean 49 years
- discount rate equal to 3%

Therefore, the PDV of someone without college education will be

$$PDV(a) = 40000 \left[\frac{1 - \left(\frac{1}{1+0.03}\right)^{49}}{1 - \left(\frac{1}{1+0.03}\right)} \right]$$

$$= 1.050 \text{ millions}$$

(b) Going to university means a higher income and the need to pay tuition fees of 20000 per year (4 years).

b(i) The PDV of the costs associated with the tuition fees are give by:

$$PDV_{\substack{(fees)\\ t=0}} = 20000 \left[\frac{1 - \left(\frac{1}{1+0.03}\right)^4}{1 - \left(\frac{1}{1+0.03}\right)} \right]$$

$$= 76572.$$

b(ii) The PDV of the income stream for 45 years is given by two steps:

$$PDV\binom{income}{t=4} = 70000 \left[\frac{1 - \left(\frac{1}{1+0.03}\right)^{45}}{1 - \left(\frac{1}{1+0.03}\right)} \right]$$

$$= 1767799$$

then we have to discount this value 4 periods backwards to period 0. That is

$$PDV\binom{income}{t=0} = \frac{PDV(income;\ t=4)}{(1+0.03)^4}$$

$$= 1570667.$$

Now, it is easy to see what is the value of human capital of going to university

$$PDV(income;\ t=0) = 1570667$$

$$-\ PDV(fees;\ t=0) = 76572$$

$$= 1494095$$

And we may conclude that:

Going to university:	1494095
Not Going to university:	1050000

(e) The economic value of education is the premium that you receive by going to university instead of not going.

$\boxed{\text{close to 44 4000 dollars.}}$