# An Overview of Long-Run Economic Growth

## — Week 3 —

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# Summary


1. Growth over the Very Long Run
2. Modern Economic Growth
3. Modern Growth around the World
4. Some Useful Properties of Growth Rates
5. The Costs of Economic Growth
6. Required reading

# I – Growth over the Very Long Run

# The Great Divergence

1. Sustained increases in standards of living are a recent phenomenon.
2. The Great Divergence
    1. The recent era of increased difference in standards of living across countries.
3. Before 1700
    1. Per capita GPD in nations differed only by a factor of two or three.
4. Today
    1. Per capita GPD differs by a factor of 50 for several countries.

# Graph of the Great Divergence

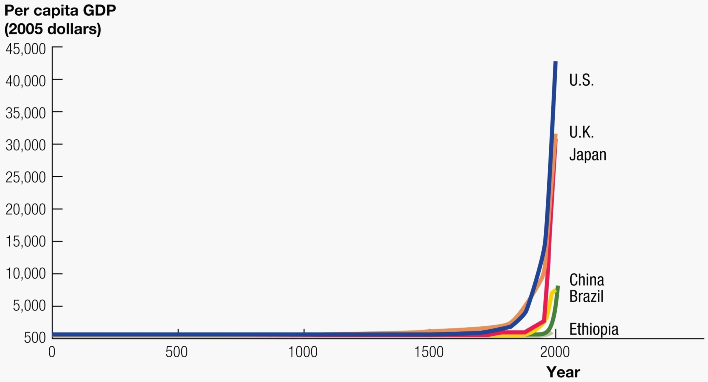


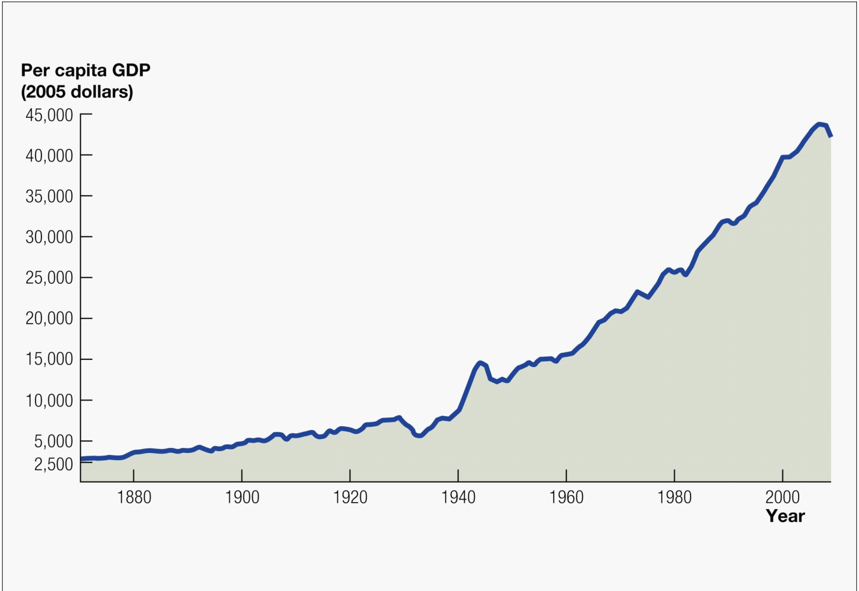

**FIGURE 3.1 Economic Growth over the Very Long Run in Six Countries**

# II – Modern Economic Growth

# The amazing improvement in living standards

1. From 1870 to 2000, United States per capita GDP ...
   1. ... rose by nearly 15-fold.
2. Implications for you?
   1. A typical college student today will earn a lifetime income about twice his or her parents.
3. See next figure.

# Per capita GDP in the United States




*Macroeconomics*, 2nd Ed

# The Definition of Economic Growth

1. Growth of per capita GDP
2. Percentage change in GDP between period $t$ and and $t-1$

$$g = \frac{y_t - y_{t-1}}{y_{t-1}}$$

3. Or

$$\begin{aligned} y_1 &= y_0(1+g)^1 \\ &... \\ y_t &= y_0(1+g)^t \end{aligned}$$

# The Rule of 70 and the Ratio Scale

1. If $y$ grows at a rate of $g$ percent per year, then the number of years it takes $y$ to double is approximately equal to

$$\frac{70}{g}$$

2. Small differences in growth rates result in large differences over time.
3. The time it takes to double only depends on the growth rate and not the initial value.
4. A ratio scale
   1. Its like a logarithm scale
   2. When plotted on a ratio scale, a variable that grows at a constant rate will be a straight line

# The absolute vsersus ratio scale

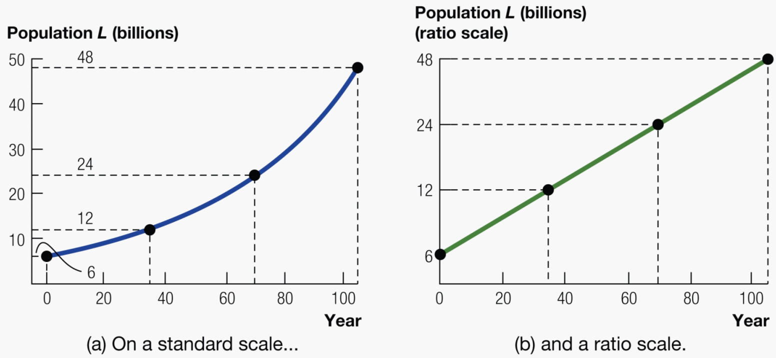



**FIGURE 3.4 Population over Time, Revisited**

# GDP has been rising since the 1880's

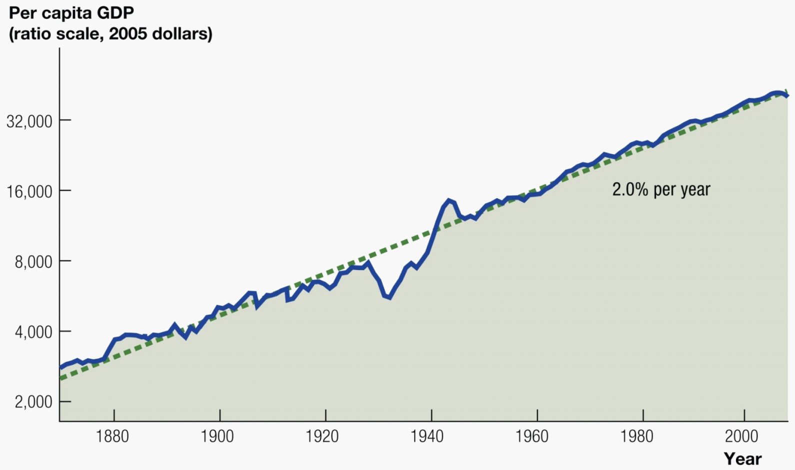



FIGURE 3.5 **Per Capita GDP in the United States, 1870–2009: Ratio Scale**

# Calculating Growth Rates

1. The rule for computing growth rates
2. This formula can be applied even if the data does not exhibit constant growth.
   1. In this case we get an **average annual growth rate**

$$y_t = y_0(1 + \bar{g})^t$$

solve for growth rate

$$\bar{g} = \left(\frac{y_t}{y_0}\right)^{1/t} - 1$$

# III – Modern Growth around the World

# Real Economic Convergence

1. After World War II, growth in Germany and Japan accelerated.
2. Convergence (real):
    1. Poorer countries will grow faster to "catch up" to the level of income in richer countries.
3. Brazil had accelerated growth until 1980 and then stagnated.
4. China and India have had the reverse pattern.

# Real convergence in a ratio scale

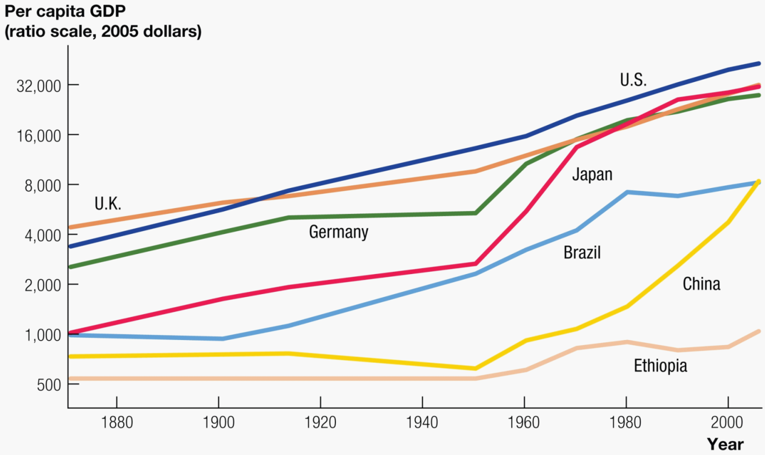
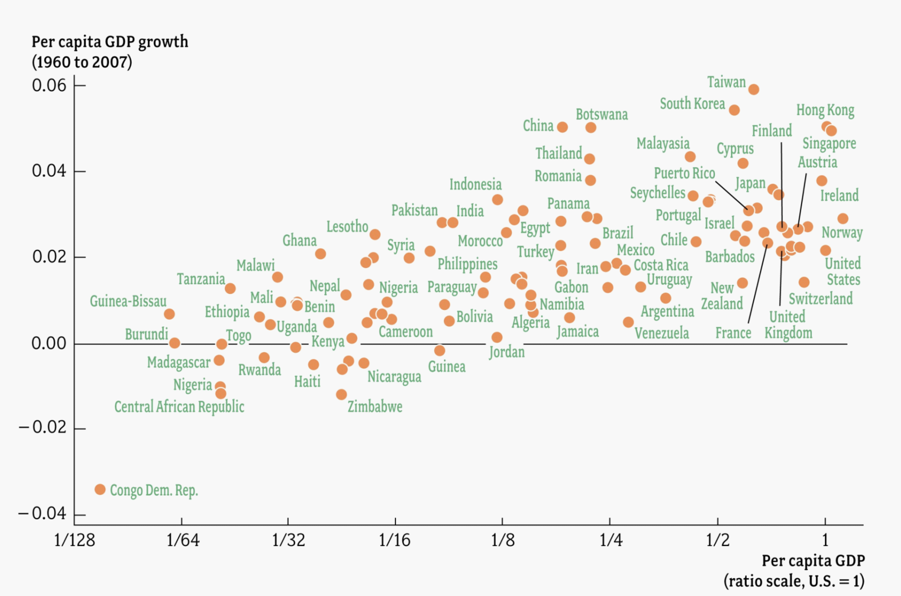



**FIGURE 3.6 Per Capita GDP in Seven Countries, 1870–2006**

# Convergence: A Broad Sample of Countries

1. Over the period 1960–2007
   1. Some countries have exhibited a negative growth rate.
   2. Other countries have sustained nearly 6 percent growth
   3. Most countries have sustained about 2 percent growth.
2. Small differences in growth rates result in large differences in standards of living.

# The lack of convergence



# IV – Some Useful Properties of Growth Rates

# Real Economic Convergence

1. Growth rates of ratios, products, and powers follow several simple rules.
2. Growth rates obey mathematical operations that are a level simpler than the operation on the original variable.
    1. Variables divided $\rightarrowtail$ growth rates subtracted
    2. Variables multiplied $\rightarrowtail$ growth rates added
    3. Variable taken to a power number $\rightarrowtail$ growth rate multiplied by that number

# Transforming functions into log-differences: summary

| *Variables in levels* | | *Variables in* $\Delta$*logs* |
|---|---|---|
| $y_t = 2x_t$ | $\Leftrightarrow$ | $g_y = g_x$ |
| $y_t = 2x_t z_t$ | $\Leftrightarrow$ | $g_y = g_x + g_z$ |
| $y_t = 2x_t z_t^{-3}$ | $\Leftrightarrow$ | $g_y = g_x - 3g_z$ |
| $y_{t+1} = x_{t+1} + z_{t+1}$ | $\Leftrightarrow$ | $g_y = g_x \left(\frac{x_t}{y_t}\right) + g_z \left(\frac{z_t}{y_t}\right)$ |
| $y_{t+1} = x_{t+1} + a$ | $\Leftrightarrow$ | $g_y = g_x \left(\frac{x_t}{y_t}\right)$ |

# An example

Suppose $x$ grows at rate $g_x = 0.10$ and $y$ grows at rate $g_y = 0.03$.
What is the growth rate of $z$ in the following cases?

| | | |
|---|---|---|
| $z = x \times y$ | $\Rightarrow$ | $g_z = g_x + g_y = .13$ |
| $z = x/y$ | $\Rightarrow$ | $g_z = g_x - g_y = .07$ |
| $z = y/x$ | $\Rightarrow$ | $g_z = g_y - g_x = -.07$ |
| $z = x^2$ | $\Rightarrow$ | $g_z = 2 \times g_x = .20$ |
| $z = y^{1/2}$ | $\Rightarrow$ | $g_z = .5 \times g_y = .015$ |
| $z = x^{1/2}y^{-1/4}$ | $\Rightarrow$ | $g_z = .5 \times g_x - .25 \times g_y = .0425$ |

# Growth Rules in a Famous Example

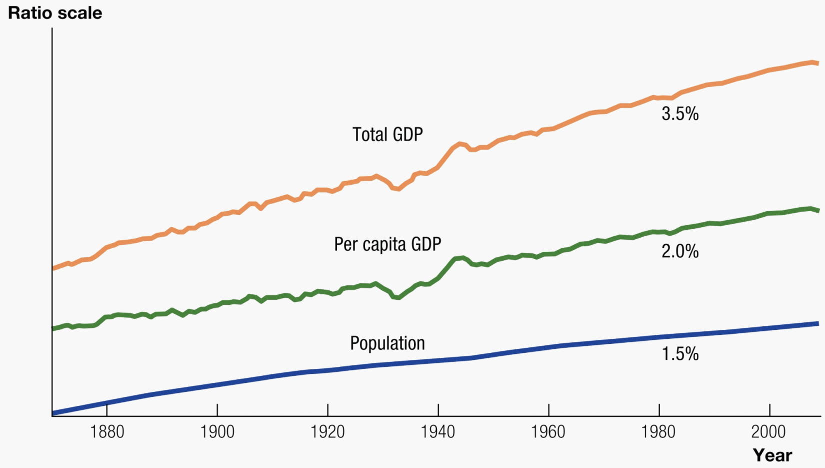
1. Original output equation

$$Y_t = A_t K_t^{1/3} L_t^{2/3}$$

2. In terms of growth rates we have

$$g_Y = g_A + (1/3)g_K + (2/3)g_L$$

# Growth Rules in a Famous Example

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# V – The Costs of Economic Growth

# The benefits of economic growth

1. Improvements in health
2. Higher incomes
3. Increase in the variety of goods and services

# The Costs of economic growth

1. Environmental problems
2. Income inequality across and within countries
3. Loss of certain types of jobs
4. Economists generally have a consensus that the benefits of economic growth outweigh the costs.

# VI – Required readings

# Required reading

For this week you are required to read **Read Chapter 3** of our adopted textbook.

Charles I. Jones (2014). Macroeconomics, Third Edition, W. W. Norton & Company.

# Appendix

# How to transform functions in levels into log differences

The knowledge of this appendix is not compulsory. This is just used as a proof of the results above.

# Transforming functions into log-differences: first case

1. **A linear function**: $Y_t = 2X_t$. Apply logs to two consecutive periods:

$$\begin{aligned} \ln Y_t &= \ln 2 + \ln X_t \\ \ln Y_{t+1} &= \ln 2 + \ln X_{t+1} \end{aligned}$$

2. Therefore, the first difference of logs is

$$\underbrace{\ln Y_{t+1} - \ln Y_t}_{\text{growth rate of } Y:\ g_Y} = (\ln 2 + \ln X_{t+1}) - (\ln 2 + \ln X_t) = \underbrace{\ln X_{t+1} - \ln X_t}_{\text{growth rate of } X:\ g_X}$$

3. In this kind of function, the growth rate of $Y$, let's call it $(g_Y)$, is equal to the growth rate of $X$, $(g_X)$

$$g_Y = g_X$$

# Transforming functions into log-differences: second case

1. **A linear function of two independent variables**: $Y_t = 2X_tZ_t$.
2. Apply logs to two consecutive periods, and you will get

$$g_Y = g_X + g_Z$$

3. Prove this result yourself.

# Transforming functions into log-differences: third case

1. **A power function**: $Y_t = 2X_t Z_t^{-3}$.
2. Apply logs

$$\begin{aligned} \ln Y_t &= \ln 2 + \ln X_t - 3\ln Z_t \\ \ln Y_{t+1} &= \ln 2 + \ln X_{t+1} - 3\ln Z_{t+1} \end{aligned}$$

3. Therefore, the first difference of logs is

$$\begin{aligned} \underbrace{\ln Y_{t+1} - \ln Y_t}_{\text{growth rate: } g_Y} &= (\ln 2 + \ln X_{t+1} - 3\ln Z_{t+1}) - (\ln 2 + \ln X_t - 3\ln Z \\ &= \underbrace{\ln X_{t+1} - \ln X_t}_{\text{growth rate: } g_X} - 3\underbrace{(\ln Z_{t+1} - \ln Z_t)}_{\text{growth rate: } g_Z} \end{aligned}$$

4. So this power function can be written in $\Delta$log as

$$g_Y = g_X - 3g_Z$$

# Transforming functions into log-differences: fourth case

1. The last function we need to consider is an **additive function** like

$$Y_{t+1} = X_{t+1} + Z_{t+1}$$

2. Here we can't apply logs. But there is another way
3. Firstly, multiply and divide through as follows

$$\frac{Y_{t+1}}{Y_t} Y_t = \frac{X_{t+1}}{X_t} X_t + \frac{Z_{t+1}}{Z_t} Z_t.$$

4. Now apply the following: $\frac{Y_{t+1}}{Y_t} = 1 + g_Y$, $\frac{X_{t+1}}{X_t} = 1 + g_X$, $\frac{Z_{t+1}}{Z_t} = 1 + g_Z$, and the previous eq. can be written as

$$(1 + g_Y) Y_t = (1 + g_X) X_t + (1 + g_Z) Z_t$$

5. Divide through by $Y_t$ and get

$$1 + g_Y = (1 + g_X) \frac{X_t}{Y_t} + (1 + g_Z) \frac{Z_t}{Y_t}$$

# Transforming functions into log-differences: fourth case (cont.)

1. Notice that the previous equation can be written as

$$1 + g_Y = \underbrace{\left(\frac{X_t}{Y_t} + \frac{Z_t}{Y_t}\right)}_{=(X_t+Z_t)/Y_t=1} + g_X\left(\frac{X_t}{Y_t}\right) + g_Z\left(\frac{Z_t}{Y_t}\right)$$

2. Therefore, an **additive function** like $Y_{t+1} = X_{t+1} + Z_{t+1}$ can be expressed as

$$g_Y = g_X\left(\frac{X_t}{Y_t}\right) + g_Z\left(\frac{Z_t}{Y_t}\right)$$

3. Notice that if $Z = 2$, its growth rate were $z = 0$, and we would get

$$g_Y = g_X\left(\frac{X_t}{Y_t}\right)$$

# Transforming functions into log-differences: summary

1. Let's summarize our results

| *Variables in levels* | | *Variables in* $\Delta$*logs* |
|---|---|---|
| $Y_t = 2X_t$ | $\Leftrightarrow$ | $g_Y = g_X$ |
| $Y_t = 2X_t Z_t$ | $\Leftrightarrow$ | $g_Y = g_X + g_Z$ |
| $Y_t = 2X_t Z_t^{-3}$ | $\Leftrightarrow$ | $g_Y = g_X - 3g_Z$ |
| $Y_{t+1} = X_{t+1} + Z_{t+1}$ | $\Leftrightarrow$ | $g_Y = g_X \left(\frac{X_t}{Y_t}\right) + g_Z \left(\frac{Z_t}{Y_t}\right)$ |
| $Y_{t+1} = X_{t+1} + a$ | $\Leftrightarrow$ | $g_Y = g_X \left(\frac{X_t}{Y_t}\right)$ |