7. Economic growth, especially in India and China, has dramatically reduced poverty in the world. In 1960, 2 out of 3 people in the world lived on less than $5 per day (in today's prices). By 2010, this number had fallen to only 1 in 12.

GROWTH RULES

The important tools we will use extensively in the coming chapters are listed below for your convenience.

- Calculating a growth rate as a percentage change: $(y_{t+1} - y_t)/y_t$.
- The constant growth rule: $y_t = y_0(1 + \bar{g})^t$ if y grows at the constant rate $\bar{g}$.
- The Rule of 70: if income grows at g percent per year, it doubles roughly every $70/g$ years.
- The ratio scale for graphs, where a variable growing at a constant rate produces a straight line.
- The formula for computing average growth rates: $g = (y_t/y_0)^{1/t} - 1$.
- The rules for computing growth rates of ratios, products, and exponentials.

1. If $z = x/y$, then $g_z = g_x - g_y$.
2. If $z = xy$, then $g_z = g_x + g_y$.
3. If $z = x^a$, then $g_z = ag_x$.

KEY CONCEPTS

constant growth rule
convergence
economic growth
the Great Divergence
growth rate
ratio scale
rule for computing growth rates
Rule of 70

REVIEW QUESTIONS

1. When and where did sustained economic growth first begin? How much inequality in per capita income was there throughout the countries of the world a thousand years ago? How much is there today?
2. How much richer is the typical 40-year-old today than the typical 40-year-old 35 years ago in the United States? What about in South Korea or China?
3. This question is not addressed in the chapter—and in fact is still debated among economists—but it is interesting to think about: Why do you suppose growth in living standards was virtually nonexistent for thousands of years? Why did this situation change in recent centuries?
4. Why are the Rule of 70 and the ratio scale useful tools? How do they work together?

5. Why, and in what sense, do the three growth rates shown in Figure 3.9 add up?
6. What are some costs and benefits of economic growth?

EXERCISES

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1. **Growth and development:** In 2010, Ethiopia had a per capita income of \$700, about \$2 per day. Compute per capita income in Ethiopia for the year 2050 assuming average annual growth is
   (a) 1% per year.
   (b) 2% per year.
   (c) 4% per year.
   (d) 6% per year.

   (For comparison, per capita income in Mexico in the year 2010 was nearly \$12,000, about 30 percent of the U.S. level.)

2. **Population growth:** Suppose the world population today is 7 billion, and suppose this population grows at a constant rate of 3% per year from now on. (This rate is almost certainly much faster than the future population growth rate; the high rate used here is useful for pedagogy. If you like, you can use a spreadsheet program to help you with this question.)
   (a) What would the population equal 100 years from now?
   (b) Compute the level of the population for $t = 0$, $t = 1$, $t = 2$, $t = 10$, $t = 25$, and $t = 50$.
   (c) Make a graph of population versus time (on a standard scale).
   (d) Now make the same graph on a ratio scale.

3. **Interest on your bank balance:** Suppose your bank account has a balance today of \$100. Consider the following time periods: $t = 0$, $t = 1$, $t = 2$, $t = 12$, $t = 24$, $t = 48$, and $t = 60$. Assume there are no deposits or withdrawals in this account other than the interest that accumulates. (If you like, use a spreadsheet program to help you with this question.)
   (a) Compute your bank balance for these time periods assuming the interest rate is 1%.
   (b) Do the same thing for an interest rate of 6%.
   (c) Plot your bank balances for these two scenarios on a standard scale.
   (d) Do the same thing with a ratio scale.

4. **Stock returns and your retirement account:** Suppose your retirement account has a balance today of \$25,000 and you are 20 years old. If you are invested in a diversified portfolio of stocks, you might hope that the historical return of about 6% continues into the future. Consider how the balance in your retirement account evolves as you age under the different assumptions below. (If you like, use a spreadsheet program to help you with this question.)
   (a) Compute the balance in your retirement account when you will be 25, 30, 40, 50, and 65 years old assuming the average annual rate of return is 6%. Assume there are no deposits or withdrawals in this account, so the original balance just accumulates.

(b) Do the same thing for rate of return of 5% and 7%. How sensitive is the calculation to the rate of return?

(c) Plot your retirement account balance for these three scenarios (6%, 5%, 7%) on a standard scale.

(d) Do the same thing with a ratio scale.

5. **The ratio scale:** Plot the following scenarios for per capita GDP on a ratio scale. Assume that per capita GDP in the year 2015 is equal to $10,000. You should not need a calculator or computer program. Use the Rule of 70 to label the value of per capita GDP on the graph in the years listed below.

(a) Per capita GDP grows at a constant rate of 5% per year between 2015 and 2085.

(b) Per capita GDP grows at 2% per year between 2015 and 2085, speeds up to 7% per year for the next 20 years, and then slows down to 5% per year for the next 28 years.

(c) Per capita GDP grows at 7% per year for 50 years and then slows down to 1% per year for the next 140 years.

6. **U.S. growth:** On page 50, we noted that the growth rate of per capita GDP in the United States between 1870 and 1929 was slightly lower than 2.0%, while the growth rate between 1950 and 2012 was slightly higher. Using the following table, calculate the actual average annual growth rates during these two periods.

Year	U.S. income
1870	$2,840
1929	$8,020
1950	$13,225
2012	$43,238

7. **Growth rates of per capita GDP:** Compute the average annual growth rate of per capita GDP in each of the cases below. The levels are provided for 1980 and 2010, measured in constant 2005 dollars.

	1980	2010
United States	24,952	41,365
Canada	23,583	37,104
Germany	21,683	34,089
France	21,441	31,299
Italy	19,554	28,377
Japan	18,749	31,447
United Kingdom	16,649	34,268
Ireland	14,642	34,877
Mexico	10,208	11,939
Brazil	6,960	8,324
Indonesia	1,500	3,966
Kenya	1,141	1,247
India	1,028	3,477
China	563	7,130
Ethiopia	466	680

8. **The costs of economic growth?** In addition to the benefits of economic growth, there are also potentially costs. What are some of these costs? Write a paragraph arguing that the benefits exceed the costs. Write a paragraph arguing the opposite, that the costs exceed the benefits. Which argument do you find more convincing, and why?

9. **Computing growth rates (I):** Suppose $x_t = (1.04)^t$ and $y_t = (1.02)^t$. Calculate the growth rate of z_t in each of the following cases:

   (a) $z = xy$
   (b) $z = x/y$
   (c) $z = y/x$
   (d) $z = x^{1/2}y^{1/2}$
   (e) $z = (x/y)^2$
   (f) $z = x^{-1/3}y^{2/3}$

10. **Computing growth rates (II):** Suppose k, l, and m grow at constant rates given by $\bar{g}_k$, $\bar{g}_l$, and $\bar{g}_m$. What is the growth rate of y in each of the following cases?

    (a) $y = k^{1/3}$
    (b) $y = k^{1/3}l^{2/3}$
    (c) $y = mk^{1/3}l^{2/3}$
    (d) $y = mk^{1/4}l^{3/4}$
    (e) $y = mk^{3/4}l^{1/4}$
    (f) $y = (klm)^{1/2}$
    (g) $y = (kl)^{1/4}(1/m)^{3/4}$

11. **Computing levels:** Suppose x_t grows at 2% per year and y_t grows at 5% per year, with $x_0 = 2$ and $y_0 = 1$. Calculate the numerical values of z_t for $t = 0$, $t = 1$, $t = 2$, $t = 10$, $t = 17$, and $t = 35$ for the following cases:

    (a) $z = x$
    (b) $z = y$
    (c) $z = x^{3/4}y^{1/4}$

12. **(Harder) An alternative way of computing growth rates?** In exercise 6, the correct way to compute the average growth rate is to apply equation (3.9). An alternative—but incorrect—way is to take the percentage change divided by the number of years: for example,

$$\frac{1}{T} \times \frac{y_T - y_0}{y_0}.$$

    Compute the growth rates with this formula; they should be substantially different from what you got before. What is the explanation for this difference?

13. **How can we measure growth over the very long run?** The poorest countries in the world have a per capita income of about \$600 today. We can reasonably assume that it is nearly impossible to live on an income below half this level (below \$300). Per capita income in the United States in 2010 was about \$43,000. With this information in mind, consider the following questions.

    (a) For how long is it possible that per capita income in the United States has been growing at an average annual rate of 2% per year?

(b) Some economists have argued that growth rates are mismeasured. For example, it may be difficult to compare per capita income today with per capita income a century ago when so many of the goods we can buy today were not available *at any price* then. Suppose the true growth rate in the past century was 3% per year rather than 2%. What would the level of per capita income in 1800 have been in this case? Is this answer plausible?

WORKED EXERCISES

3. Interest on your bank balance:

(a) To calculate your bank balance in any period, we use the formula from the constant growth rule in equation (3.7) on page 48:

$$y_t = y_0(1 + \bar{g})^t.$$

Let B_t denote the bank balance and $\bar{r}$ denote the interest rate. Then your bank balance satisfies

$$B_t = B_0(1 + \bar{r})^t.$$

With B_0 = \$100 and $\bar{r} = 0.01$, the bank balances are shown in the second column of Table 3.2. For example, after 60 years, the bank balance has reached \$182, not quite double the original value (recall that according to the Rule of 70, it will take 70 years for the bank balance to double if the interest rate is 1%).

(b) We use the same formula with $\bar{r} = 0.06$ to calculate the bank balances with a 6% interest rate. These are shown in the last column of Table 3.2. Notice that the Rule of 70 applies here as well, so the bank balance doubles about every 12 years. (This is why those somewhat odd-looking times were chosen, rather than $t = 5$, $t = 10$, etc.) Notice how the seemingly small difference in interest rates—1% versus 6%—turns into enormous differences

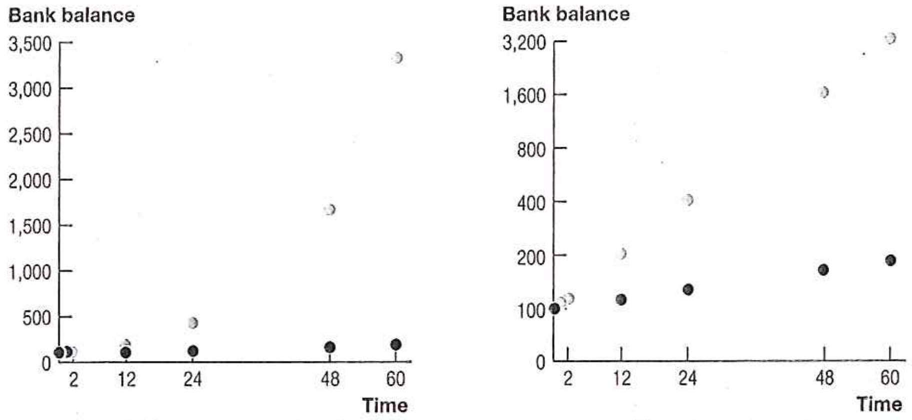
TABLE 3.2

Bank Balances

Time	Interest rate $\bar{r} = 0.01$	Interest rate $\bar{r} = 0.06$
0	100	100
1	101	106
2	102	112
12	113	201
24	127	405
48	161	1,639
60	182	3,299

FIGURE 3.10

Bank Balances



(a) On a standard scale... (b) and a ratio scale.

in your bank balance. After 60 years, the balance is nearly \$3,300 when the interest rate is 6%.

(c) and (d) Figure 3.10 shows the bank balances on a standard scale and a ratio scale (also called a "logarithmic scale" in some spreadsheet programs). On the standard scale, we see the data points curve upward, following the standard pattern of economic growth. Because the interest rate is constant, the upward curve turns into a straight line on the ratio scale. This is what you need to keep in mind when you are making the plot on the ratio scale: if you are not using a spreadsheet program, you just draw the straight line and label the points as in the table.

6. **U.S. growth:** To compute the average annual growth rate, we use the formula from equation (3.9) on page 52:

$$\bar{g} = \left(\frac{y_t}{y_0}\right)^{1/t} - 1.$$

Notice that this formula is derived from the familiar expression $y_t = y_0(1 + \bar{g})^t$.

For the period 1870 to 1929, the formula yields a growth rate of

$$\bar{g} = \left(\frac{8{,}020}{2{,}840}\right)^{1/(1929-1870)} - 1 = 0.0178.$$

So the growth rate in this initial period averaged 1.78% per year.

For the period 1950 to 2010, the formula yields a growth rate of

$$\bar{g} = \left(\frac{43{,}238}{13{,}225}\right)^{1/(2010-1950)} - 1 = 0.0199.$$

So the growth rate in the more recent period averaged 1.99% per year.